LAST NAME: ________

FIRST NAME: ________

Problem 1 **(a)** Calculate the image of the sequence $\langle 3, 0, 2\rangle$ under Gödel numbering and show your work. If this image does not exist, prove it.

Answer:

$2^{3+1} \cdot 3^{0+1} \cdot 5^{2+1} =$

$2^4 \cdot 3 \cdot 5^3 = 10^3 \cdot 6 =$

$\boxed{6000}$

(b) Calculate the pre-image of the number 2940 under Gödel numbering and show your work. If this pre-image does not exist, prove it.

Answer:

$2940 = 10 \cdot 294 =$

$10 \cdot 3 \cdot 98 =$

$2 \cdot 3 \cdot 5 \cdot 2 \cdot 49 =$

$2^2 \cdot 3^1 \cdot 5^1 \cdot 7^2$

answer:

$\boxed{\langle 1, 0, 0, 1\rangle}$

(c) Calculate the pre-image of the number 3850 under Gödel numbering and show your work. If this pre-image does not exist, prove it.

Answer:

$3850 = 385 \cdot 10 =$

$2 \cdot 5 \cdot 7 \cdot 55 =$

$2 \cdot 5^2 \cdot 7 \cdot 11$

not a Gödel number, misses 3, no pre-image.

(d) Let m be the image of the sequence

$$s = \langle x_1, x_2, x_3, x_4, x_5, x_6\rangle$$

(with 2 3 5 7 11 13 written above the components)

under Gödel numbering. Represent the pre-image of the number $78\,m$ as a function of the (components of) sequence s. If such a representation does not exist, prove it.

Answer:

$78m = 6 \cdot 13 =$

$= 2 \cdot 3 \cdot 13$

answer:

$\boxed{\langle x_1+1, x_2+1, x_3, x_4, x_5, x_6+1\rangle}$

(e) Let n be the image of the sequence

$$\langle x_1, x_2, x_3, x_4\rangle$$

under Gödel numbering. Represent the image of the sequence

$$\langle x_1{+}2,\ x_2{+}1,\ x_3,\ x_4,\ 1\rangle$$

(with 2 3 5 7 11 written above the components)

as a function of n. If such a representation does not exist, prove it.

Answer:

answer

$n \cdot 2^2 \cdot 3^1 \cdot 11^2 =$

$= 12 \cdot 121 n$

$\boxed{= 1452 n}$

LAST NAME: ____________________

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Problem 1 **(a)** Calculate the image of the sequence $\langle 5, 0, 1\rangle$ under Gödel numbering and show your work. If this image does not exist, prove it.

Answer:

$2^{5+1}\cdot 3^{0+1}\cdot 5^{1+1}=$

$2^{6}\cdot 3\cdot 5^{2}=10^{2}\cdot 3\cdot 16$

$=\boxed{4800}$

(b) Calculate the pre-image of the number 2730 under Gödel numbering and show your work. If this pre-image does not exist, prove it.

Answer:

$2730=10\cdot 273=$

$2\cdot 5\cdot 3\cdot 91=2\cdot 5\cdot 7\cdot 13$

$2\cdot 3\cdot 5\cdot 7\cdot 13$

misses 11, not a Gödel number, no pre-image

(c) Calculate the pre-image of the number 6930 under Gödel numbering and show your work. If this pre-image does not exist, prove it.

Answer:

$6930=10\cdot 693=$

$=10\cdot 9\cdot 77=$

$=2\cdot 5\cdot 3^{2}\cdot 7\cdot 11=$

$=2^{1}\cdot 3^{2}\cdot 5^{1}\cdot 7^{1}\cdot 11^{1}$

answer: $\boxed{\langle 0,1,0,0,0\rangle}$

(d) Let n be the image of the sequence

$$\langle x_1, x_2, x_3, x_4\rangle$$

under Gödel numbering. Represent the image of the sequence

$$\langle \overset{2}{x_1+1},\ \overset{3}{x_2},\ \overset{5}{x_3+1},\ \overset{7}{x_4},\ \overset{11}{2}\rangle$$

as a function of n. If such a representation does not exist, prove it.

Answer:

$n\cdot 2\cdot 5\cdot 11^{3}$

$=10n\cdot 11\cdot 121$

$=10n\cdot 1331$

$=\boxed{13310n}$

(e) Let m be the image of the sequence

$$s=\langle \overset{2}{x_1}, \overset{3}{x_2}, \overset{5}{x_3}, \overset{7}{x_4}, \overset{11}{x_5}, \overset{13}{x_6}\rangle$$

under Gödel numbering. Represent the pre-image of the number $143\,m$ as a function of the (components of) sequence s. If such a representation does not exist, prove it.

Answer:

$143=13\cdot 11$

answer:

$\langle x_1, x_2, x_3, x_4, x_5+1, x_6+1\rangle$

Problem 2 Let L be the language defined by the regular expression:

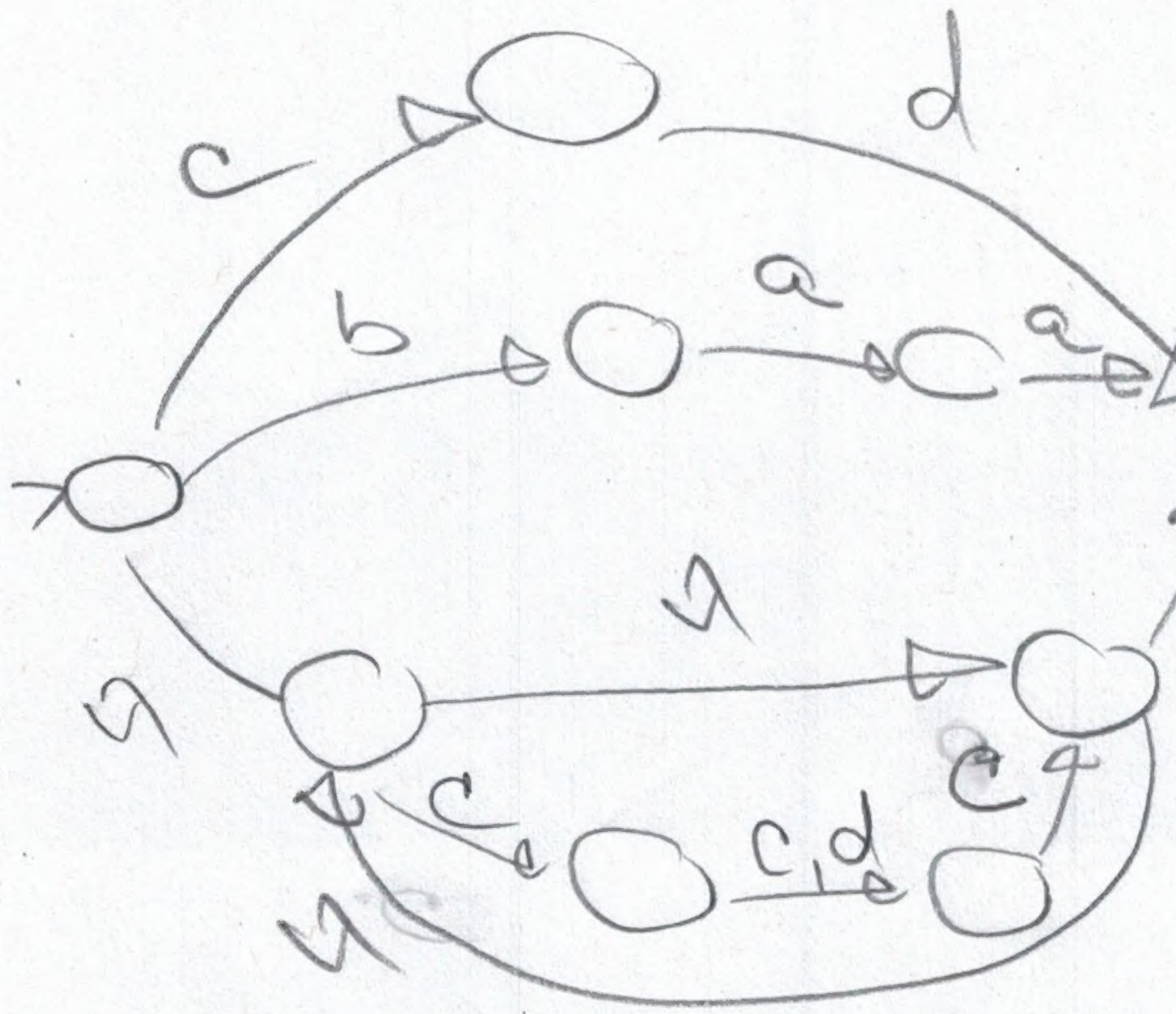
LAST NAME: ______________________

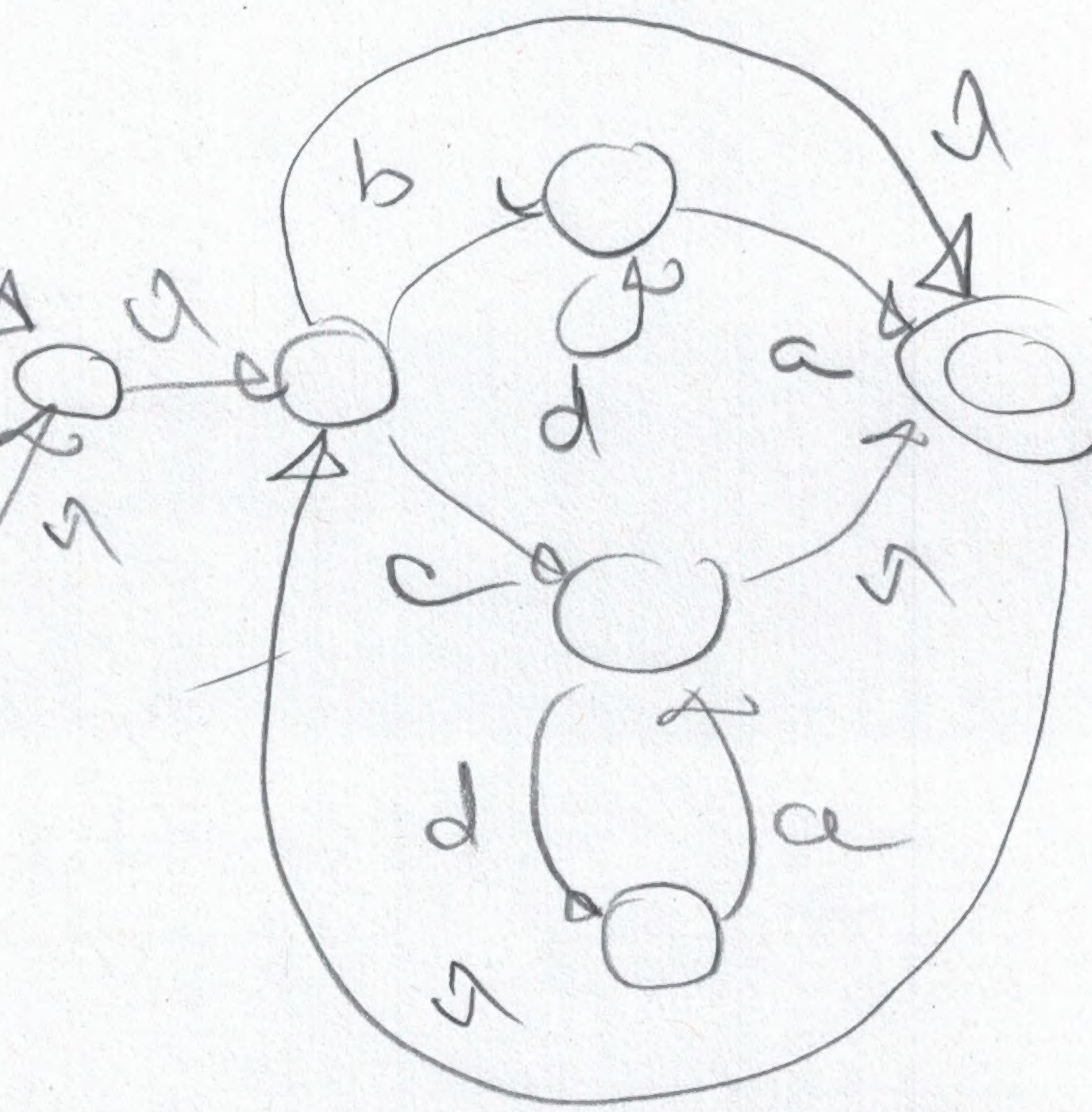
FIRST NAME: ______________________

$$(\boldsymbol{cd} \cup \boldsymbol{baa} \cup (\boldsymbol{c}\,(\boldsymbol{c} \cup \boldsymbol{d})\,\boldsymbol{c})^*)\,(\boldsymbol{c}\,(\boldsymbol{da})^* \cup \boldsymbol{bd^*a})^*$$

(a) Draw a state-transition graph of a finite automaton that accepts the language L. If such an automaton does not exist, state it and explain why.

Answer:





(b) Write a complete formal definition of a context-free grammar that generates the language L. If such a grammar does not exist, state it and explain why.

Answer:

$G = (V, \Sigma, P, S)$

$V = \{S, A, D, B, E, F\}$, $\Sigma = \{a, b, c, d\}$

P: $S \to AB$

$A \to cd \mid baa \mid D$

$D \to \lambda \mid DD \mid ccc \mid cdc$

$B \to \lambda \mid BB \mid cE \mid bFa$

$E \to daE \mid \lambda$

$F \to bF \mid \lambda$

(c) State the cardinality of the set L. (If L is a finite set, state the exact number of elements of L. Otherwise, state that L is infinite and specify whether it is countable or not.)

Answer:

L is infinite and countable.

LAST NAME: ____________

FIRST NAME: ____________

Problem 2 Let L be the language defined by the regular expression:

$$(a\,(b \cup c)^* \cup db^*c)^*\,(ab \cup dcc \cup (abaa)^*)$$

(a) Write a complete formal definition of a context-free grammar that generates the language L. If such a grammar does not exist, state it and explain why.

Answer:

$G = (V, \Sigma, P, S)$

$V = \{S, A, B, D, E, H\}$

$\Sigma = \{a, b, c, d\}$

P: $S \to AB$

$A \to \lambda \mid AA \mid aD \mid dEc$ $\quad H \to \lambda \mid HH \mid abac$

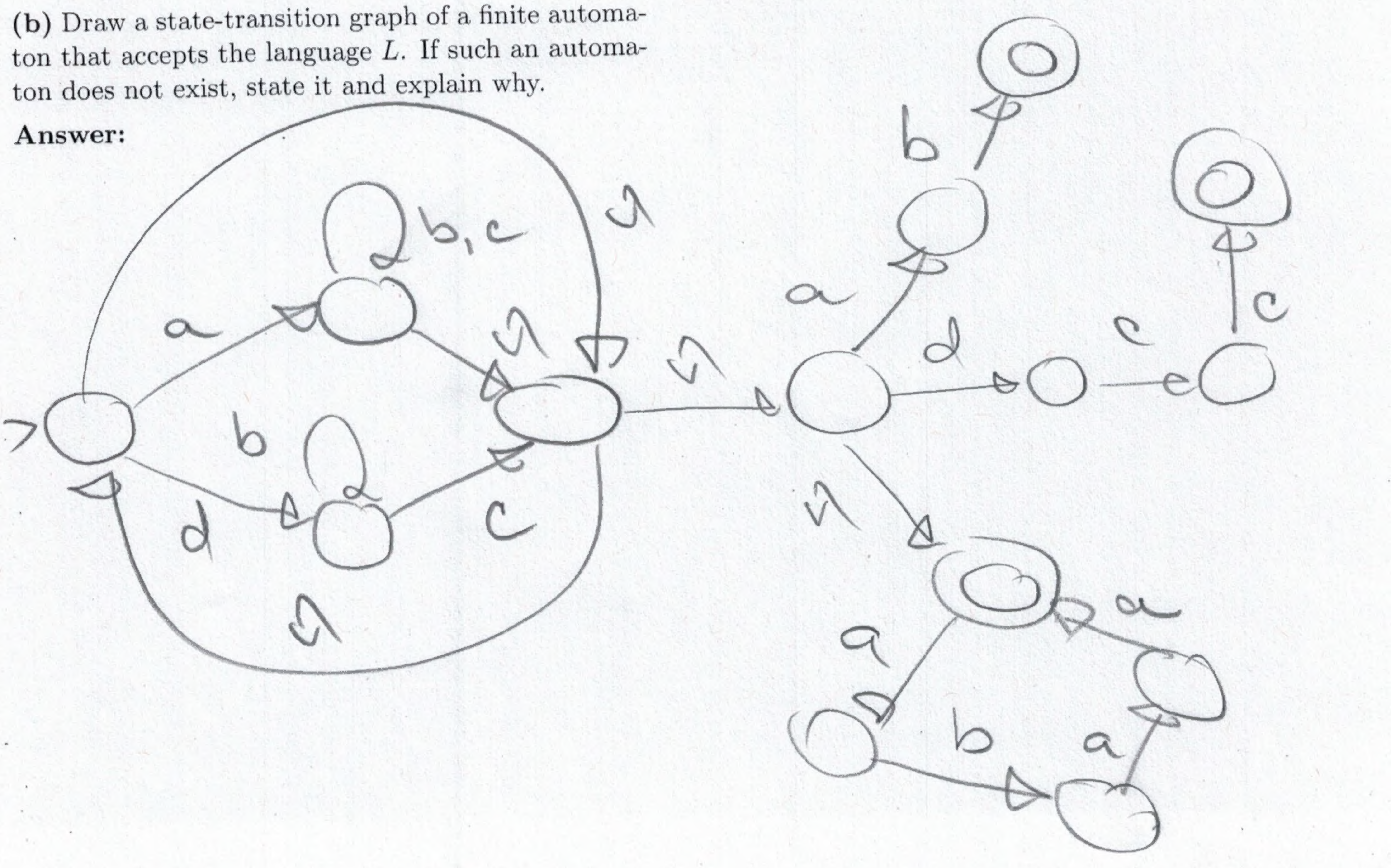
$D \to \lambda \mid DD \mid b \mid c$

$E \to bE \mid \lambda$

$B \to ab \mid dcc \mid H$

(b) Draw a state-transition graph of a finite automaton that accepts the language L. If such an automaton does not exist, state it and explain why.

Answer:



(c) State the cardinality of the set L. (If L is a finite set, state the exact number of elements of L. Otherwise, state that L is infinite and specify whether it is countable or not.)

Answer:

L is infinite and countable

LAST NAME: ______________________

FIRST NAME: ______________________

Problem 3 Let L_1 be the set of exactly those strings over the alphabet $\{a, b, c\}$ whose length is not greater than 3.

Let L_2 be the set of exactly those strings over the alphabet $\{a, b, c\}$ where the number of a's is not less than 2.

(a) Write a regular expression that represents the language L_1. If such a regular expression does not exist, state it and explain why.

Answer:

$(a \cup b \cup c \cup \Lambda)(a \cup b \cup c \cup \Lambda)(a \cup b \cup c \cup \Lambda)$

(b) Write a regular expression that represents the language L_2. If such a regular expression does not exist, state it and explain why.

Answer:

$(b \cup c)^* a (b \cup c)^* a (a \cup b \cup c)^*$

(c) Write a regular expression that represents the language $L_1 \cup L_2$. If such a regular expression does not exist, state it and explain why.

Answer:

$(a \cup b \cup c \cup \Lambda)(a \cup b \cup c \cup \Lambda)(a \cup b \cup c \cup \Lambda) \cup$
$(b \cup c)^* a (b \cup c)^* a (a \cup b \cup c)^*$

(d) Write a regular expression that represents the language $L_1 L_1$. If such a regular expression does not exist, state it and explain why.

Answer:

$(a \cup b \cup c \cup \Lambda)(a \cup b \cup c \cup \Lambda)(a \cup b \cup c \cup \Lambda)(a \cup b \cup c \cup \Lambda)\cdot$
$(a \cup b \cup c \cup \Lambda)(a \cup b \cup c \cup \Lambda)$

(e) State the cardinality of the set L_1. (If L_1 is finite set, state the exact number of elements of L_1. Otherwise, state that L_1 is infinite and specify whether it is countable or not.)

Answer: $1 + 3 + 9 + 27 = \boxed{40}$

(f) State the cardinality of the set L_2. (If L_2 is a finite set, state the exact number of elements of L_2. Otherwise, state that L_2 is infinite and specify whether it is countable or not.)

Answer: infinite and counteble

(g) State the cardinality of the set L_1^*. (If L_1^* is a finite set, state the exact number of elements of L_1^*. Otherwise, state that L_1^* is infinite and specify whether it is countable or not.)

Answer: infinite and counteble

(h) State the cardinality of the set $\mathcal{P}(L_2)$ (set of subsets of L_2). (If $\mathcal{P}(L_2)$ is a finite set, state the exact number of its elements. Otherwise, state that $\mathcal{P}(L_2)$ is infinite and specify whether it is countable or not.)

Answer: infinite and uncounteble

LAST NAME: ______________________

FIRST NAME: ______________________

Problem 3 Let L_1 be the set of exactly those strings over the alphabet $\{a, b, c\}$ whose length is equal to 3 or 4.

Let L_2 be the set of exactly those strings over the alphabet $\{a, b, c\}$ where the number of c's is not less than 3.

(a) Write a regular expression that represents the language L_1. If such a regular expression does not exist, state it and explain why.

Answer:

$(a \cup b \cup c)(a \cup b \cup c)(a \cup b \cup c)(a \cup b \cup c \cup \Lambda)$

(b) Write a regular expression that represents the language L_2. If such a regular expression does not exist, state it and explain why.

Answer:

$(a \cup b)^* c (a \cup b)^* c (a \cup b)^* c (a \cup b \cup c)^*$

(c) Write a regular expression that represents the language L_1L_1. If such a regular expression does not exist, state it and explain why.

Answer:

$(a \cup b \cup c)(a \cup b \cup c)(a \cup b \cup c)(a \cup b \cup c \cup \Lambda)(a \cup b \cup c)(a \cup b \cup c) \cdot (a \cup b \cup c)(a \cup b \cup c \cup \Lambda)$

(d) Write a regular expression that represents the language $L_1 \cup L_2$. If such a regular expression does not exist, state it and explain why.

Answer:

$(a \cup b \cup c)(a \cup b \cup c)(a \cup b \cup c)(a \cup b \cup c \cup \Lambda) \cup (a \cup b)^* c (a \cup b)^* c (a \cup b)^* c (a \cup b \cup c)^*$

(e) State the cardinality of the set L_1. (If L_1 is finite set, state the exact number of elements of L_1. Otherwise, state that L_1 is infinite and specify whether it is countable or not.)

Answer: $3^3 + 3^4 = 27 + 81 = \boxed{108}$

(f) State the cardinality of the set L_2. (If L_2 is a finite set, state the exact number of elements of L_2. Otherwise, state that L_2 is infinite and specify whether it is countable or not.)

Answer: infinite and countable

(g) State the cardinality of the set $\mathcal{P}(L_2)$ (set of subsets of L_2). (If $\mathcal{P}(L_2)$ is a finite set, state the exact number of its elements. Otherwise, state that $\mathcal{P}(L_2)$ is infinite and specify whether it is countable or not.)

Answer: infinite and uncountable

(h) State the cardinality of the set L_1^*. (If L_1^* is a finite set, state the exact number of elements of L_1^*. Otherwise, state that L_1^* is infinite and specify whether it is countable or not.)

Answer: infinite and countable

LAST NAME: ________________________

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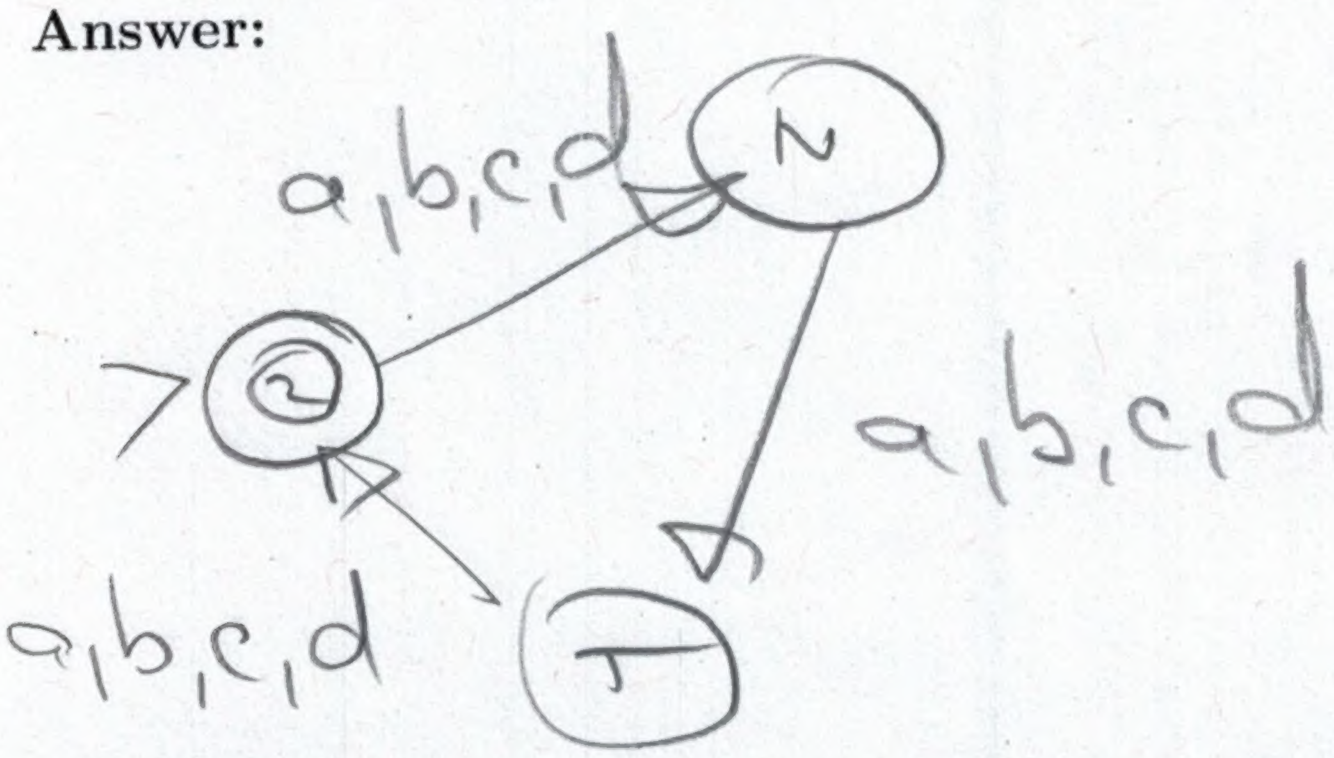
Problem 4 Let L_1 be the set of exactly those strings over the alphabet $\{a, b, c, d\}$ whose length is divisible by 3.

Let L_2 be the set of exactly those strings over the alphabet $\{a, b, c, d\}$ where the total number of c's and d's (together) is odd.

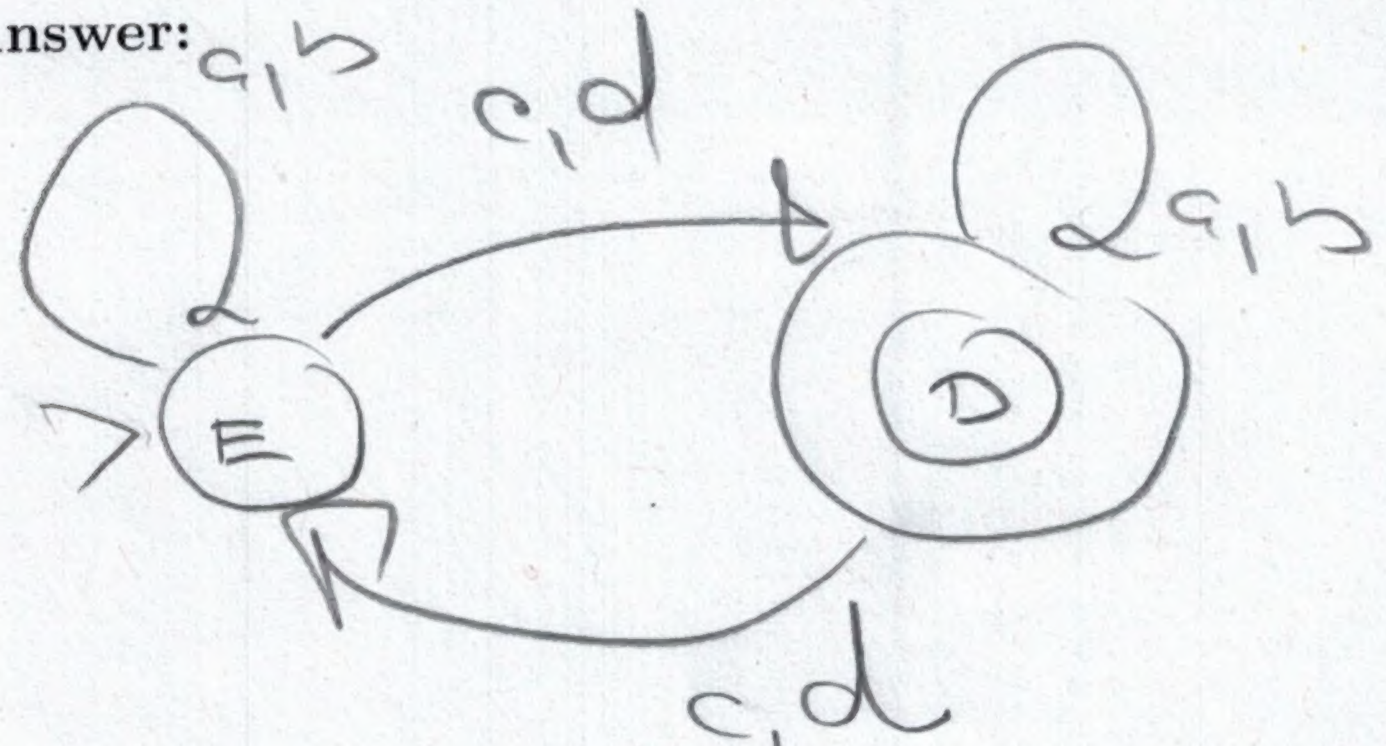
(a) Draw a state-transition graph of a finite automaton that accepts the language L_1. If such an automaton does not exist, state it and explain why.

Answer:



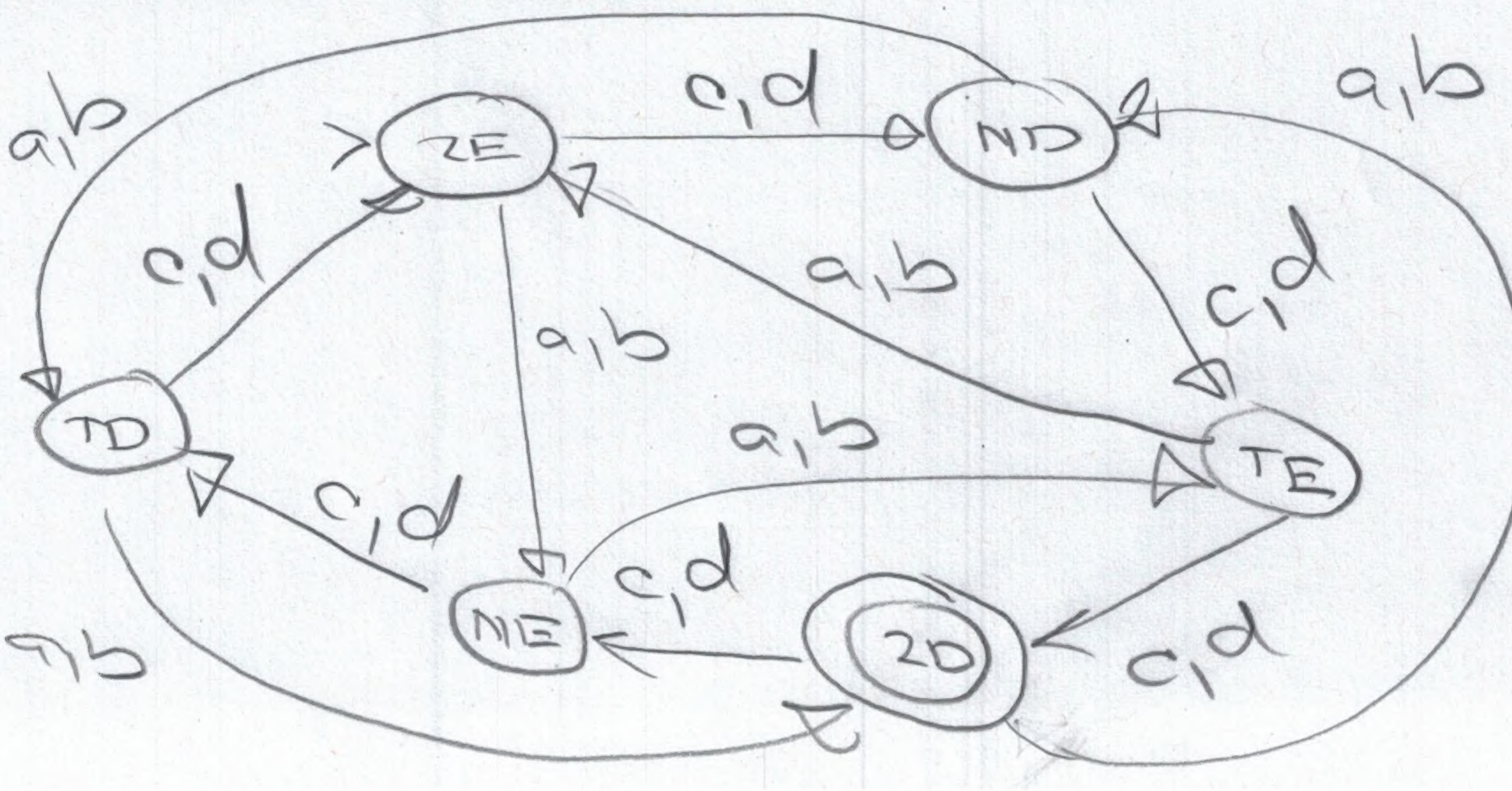
(b) Draw a state-transition graph of a finite automaton that accepts the language L_2. If such an automaton does not exist, state it and explain why.

Answer:



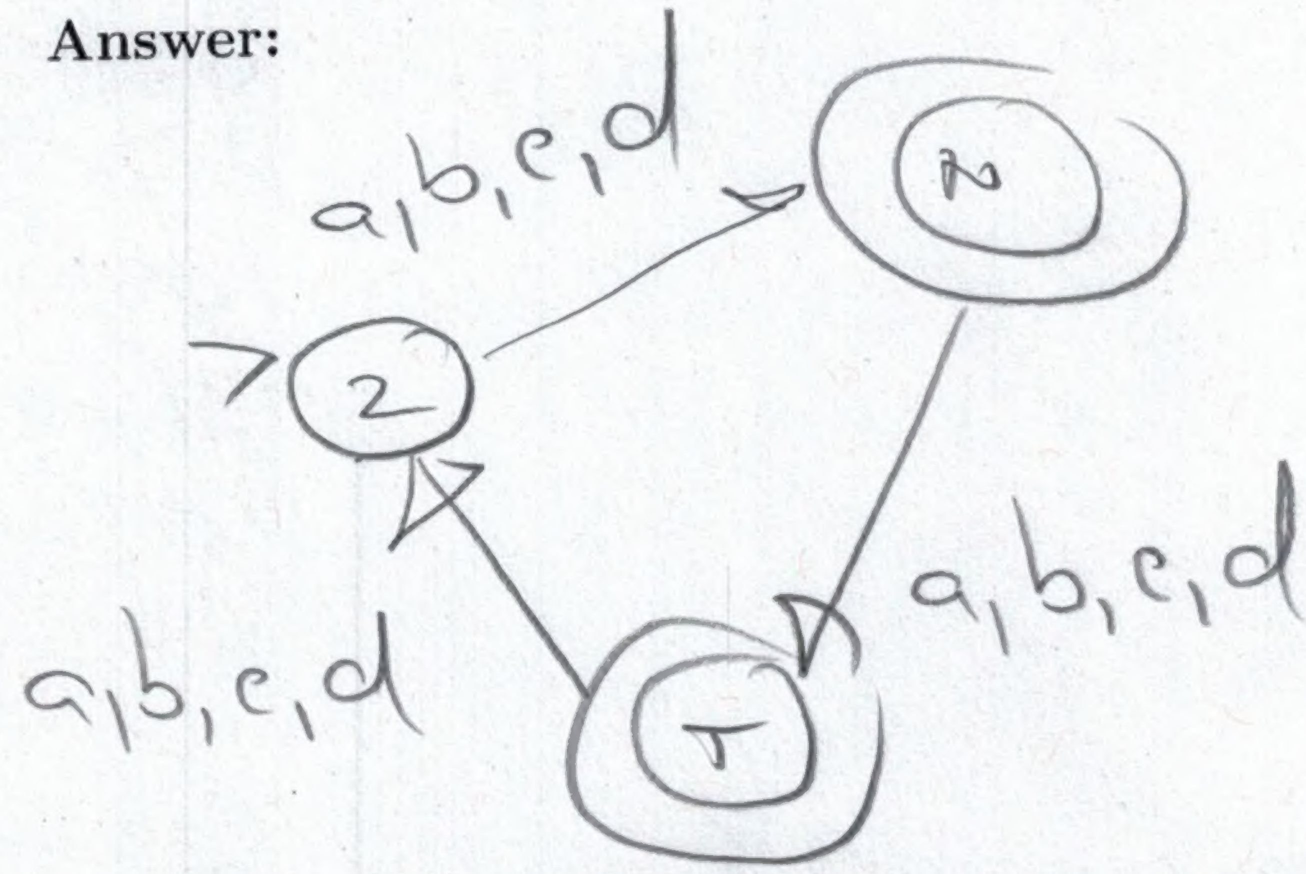
(c) Draw a state-transition graph of a finite automaton that accepts the language $L_1 \cap L_2$. If such an automaton does not exist, state it and explain why.

Answer:



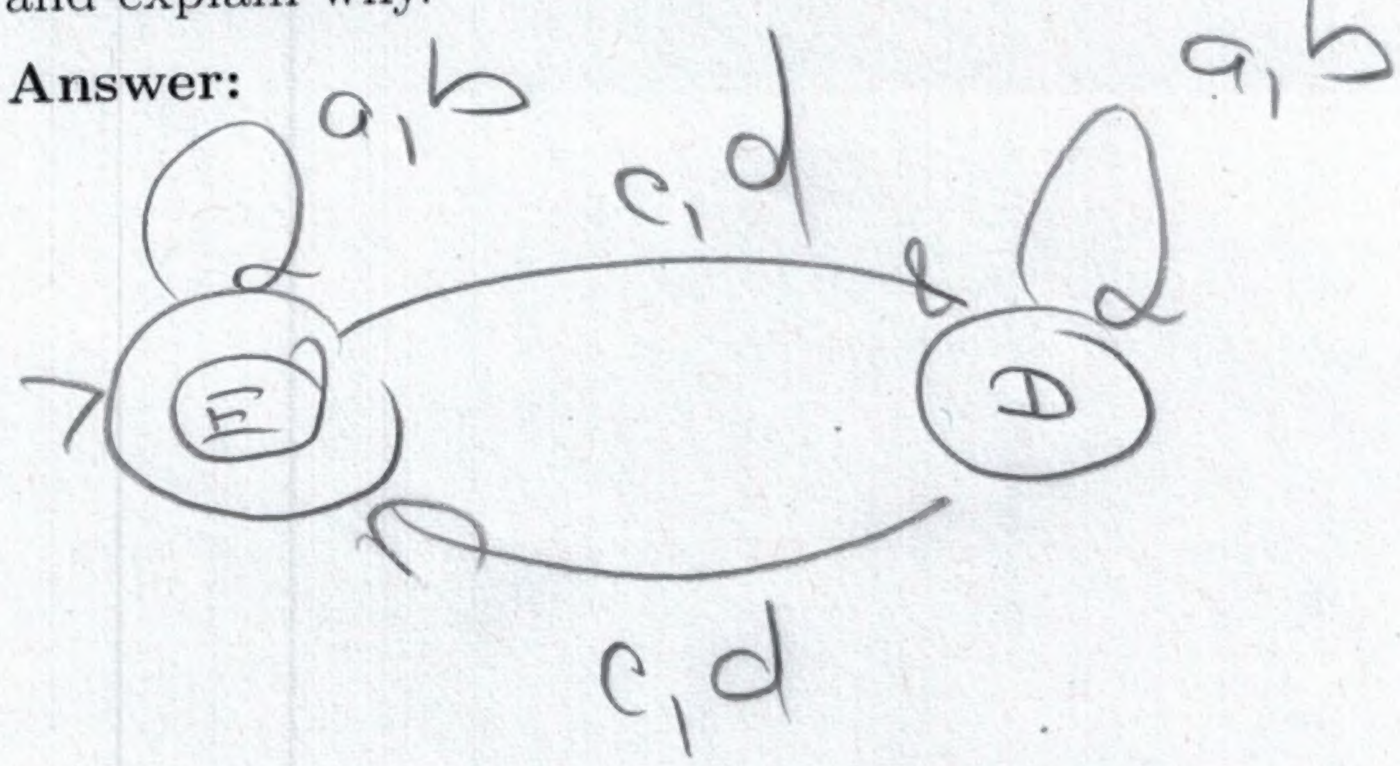
(d) Draw a state-transition graph of a finite automaton that accepts the language $\overline{L_1}$ (the complement of L_1.) If such an automaton does not exist, state it and explain why.

Answer:



(e) Draw a state-transition graph of a finite automaton that accepts the language $\overline{L_2}$ (the complement of L_2.) If such an automaton does not exist, state it and explain why.

Answer:



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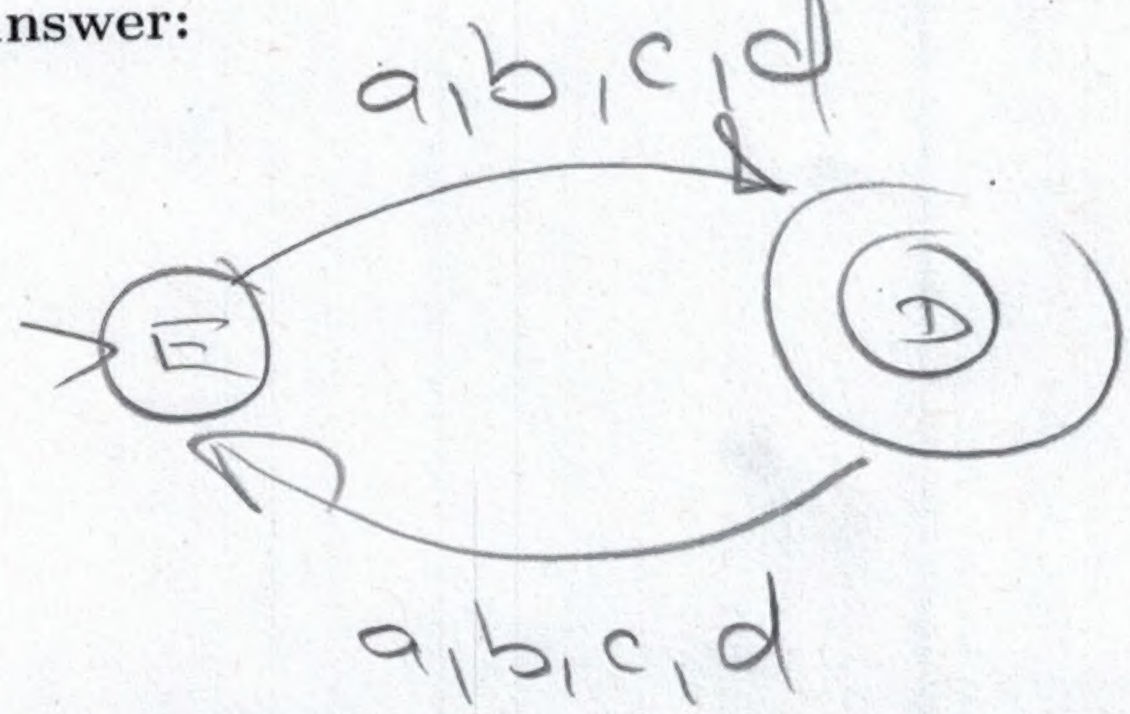
Problem 4 Let L_1 be the set of exactly those strings over the alphabet $\{a, b, c, d\}$ whose length is odd.

Let L_2 be the set of exactly those strings over the alphabet $\{a, b, c, d\}$ where the total number of a's and c's (together) is divisible by 3.

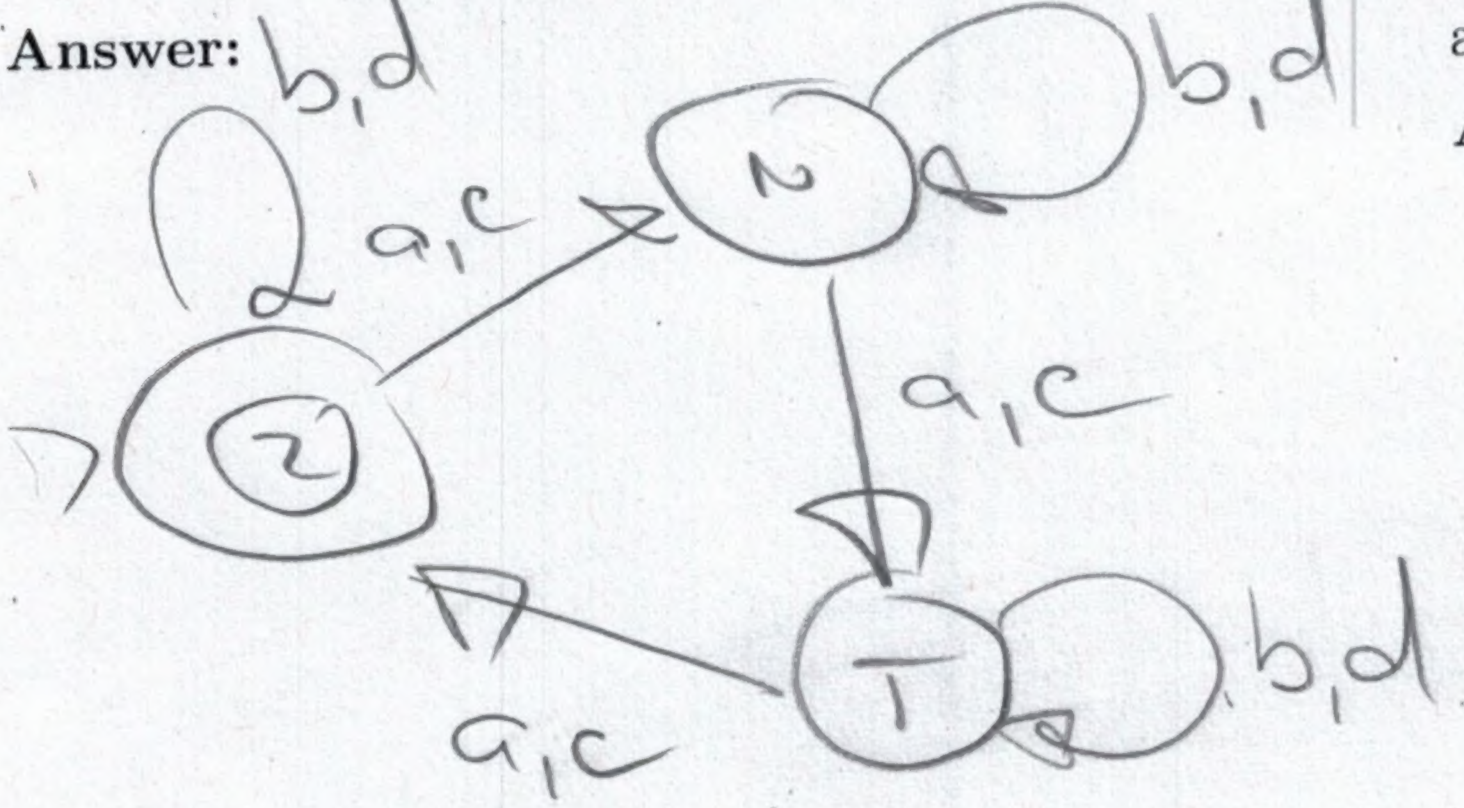
(a) Draw a state-transition graph of a finite automaton that accepts the language L_1. If such an automaton does not exist, state it and explain why.

Answer:



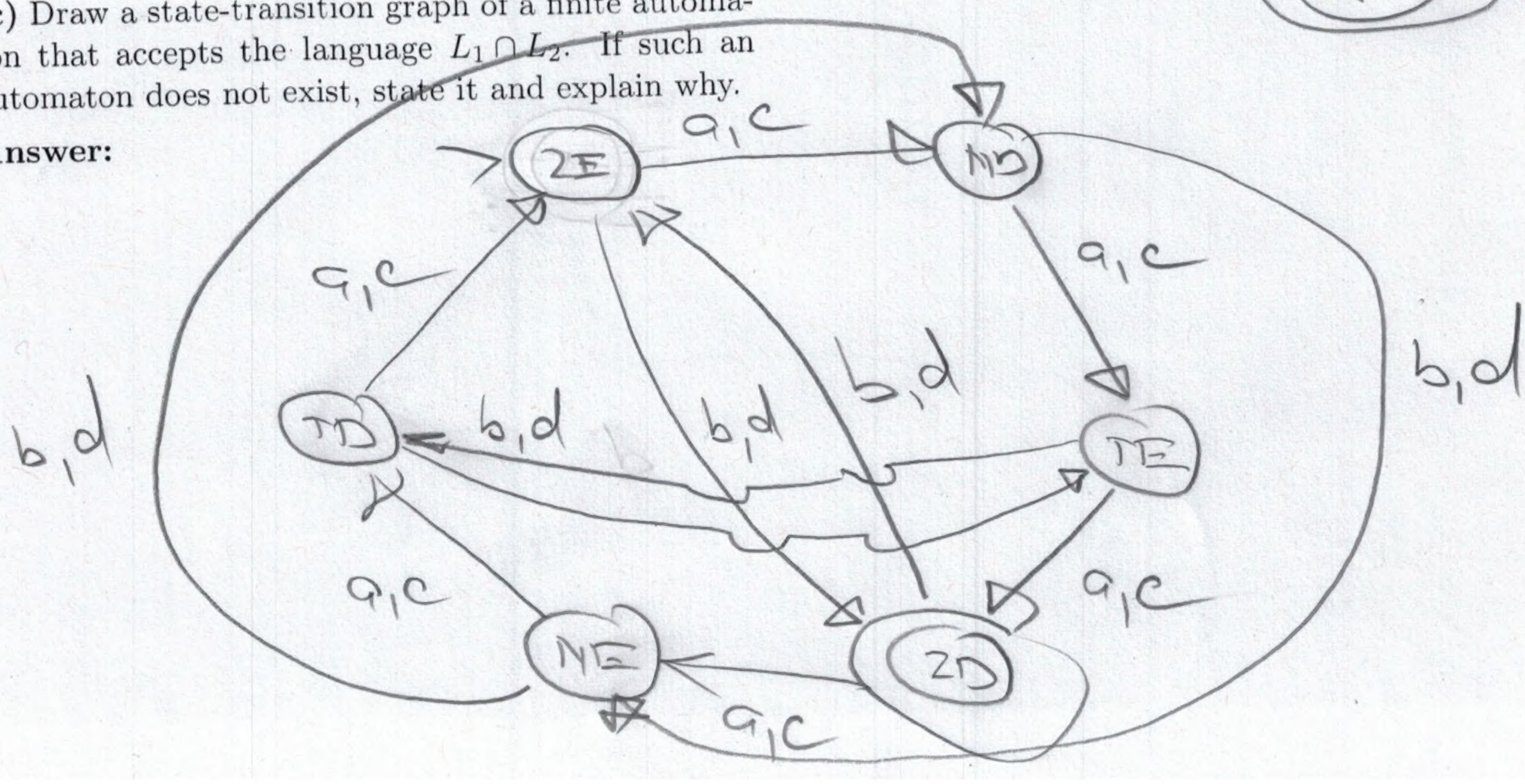
(b) Draw a state-transition graph of a finite automaton that accepts the language L_2. If such an automaton does not exist, state it and explain why.

Answer:



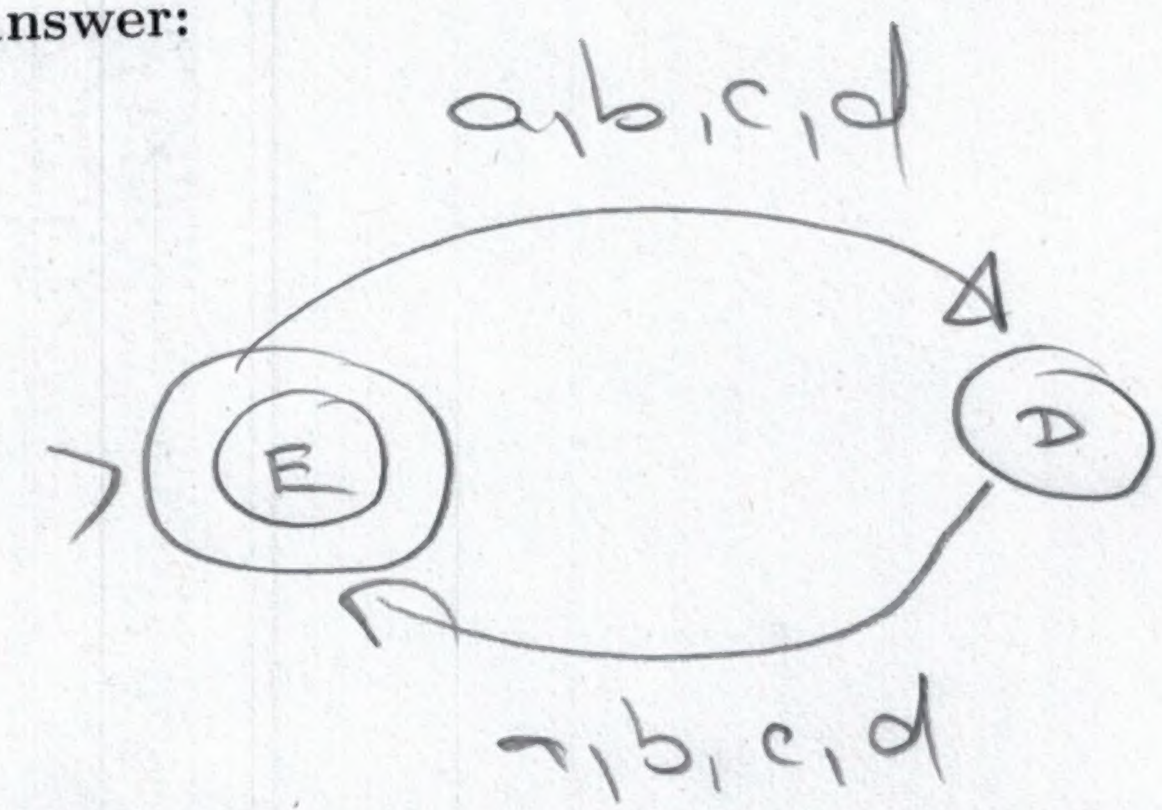
(c) Draw a state-transition graph of a finite automaton that accepts the language $L_1 \cap L_2$. If such an automaton does not exist, state it and explain why.

Answer:



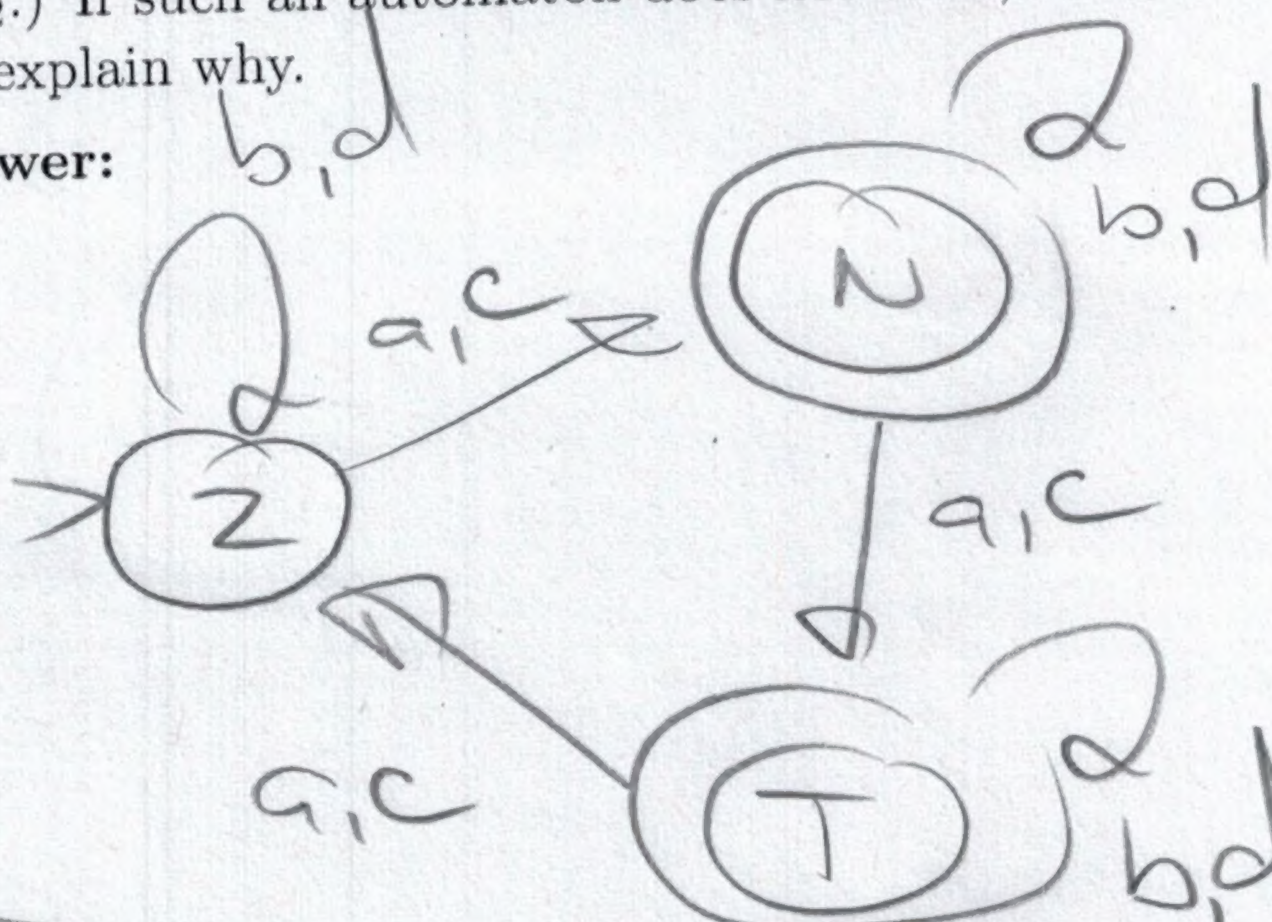
(d) Draw a state-transition graph of a finite automaton that accepts the language $\overline{L_1}$ (the complement of L_1.) If such an automaton does not exist, state it and explain why.

Answer:



(e) Draw a state-transition graph of a finite automaton that accepts the language $\overline{L_2}$ (the complement of L_2.) If such an automaton does not exist, state it and explain why.

Answer:



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Problem 5 Let L be the set of exactly those strings over the alphabet $\{a, b, c\}$ which satisfy all of the following properties:

1. begins and ends with the same letter;
2. contains exactly two c's.

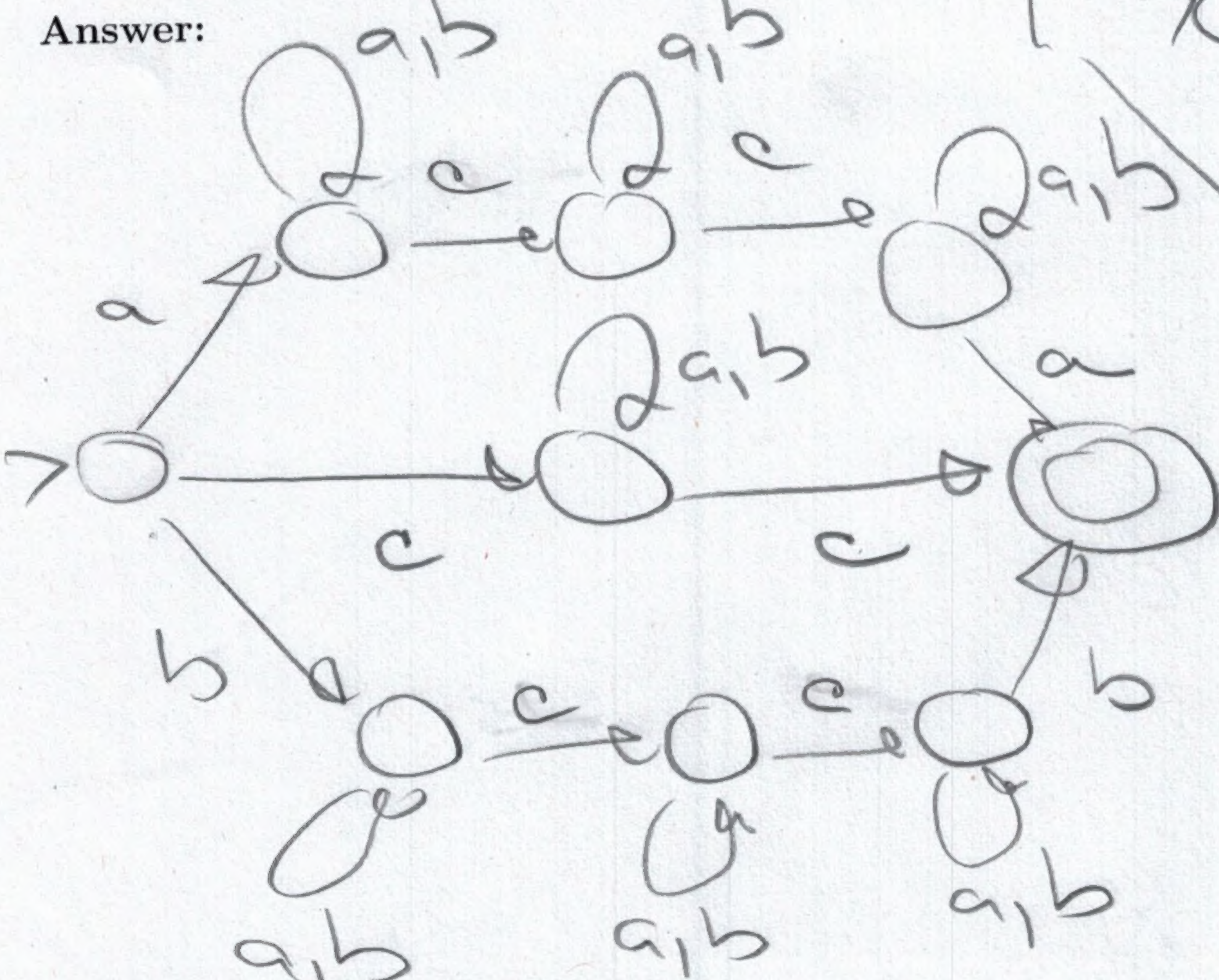
(a) Write a regular expression that represents the language L. If such a regular expression does not exist, state it and explain why.

Answer:

$$a(a \cup b)^* c (a \cup b)^* c (a \cup b)^* a$$
$$\cup$$
$$b(a \cup b)^* c (a \cup b)^* c (a \cup b)^* b$$
$$\cup$$
$$c(a \cup b)^* c$$

(b) Draw a state-transition graph of a finite automaton that accepts the language L. If such an automaton does not exist, state it and explain why.

Answer:



(c) Write a complete formal definition of a context-free grammar that generates the language L. If such a grammar does not exist, state it and explain why.

Answer:

$G = (V, \Sigma, P, S)$

$\Sigma = \{a, b, c\}$

$V = \{S, A, B, K, D\}$

P: $S \to A \mid B \mid K$

$A \to aDcDcDa$

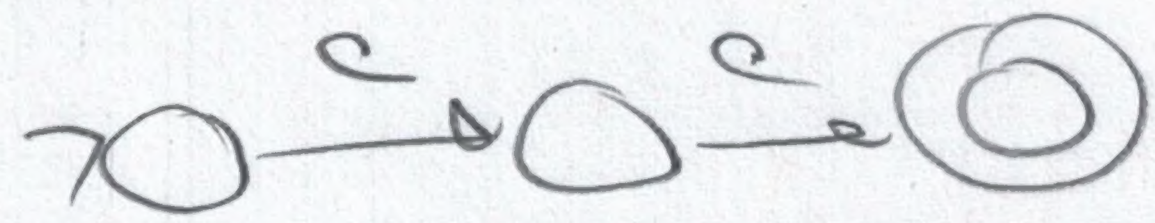
$B \to bDcDcDb$

$K \to cDc$

$D \to \varepsilon \mid DD \mid a \mid b$

(d) Draw a state-transition graph of a finite automaton that accepts the language $L \cap \mathbf{c}^*$. If such an automaton does not exist, state it and explain why.

Answer:



LAST NAME: ____________________

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Problem 5 Let L be the set of exactly those strings over the alphabet $\{a, b, c\}$ which satisfy all of the following properties:

1. first letter is either a or b;
2. last letter is either b or c;
3. first letter is different from the last letter;
4. contains exactly two c's.

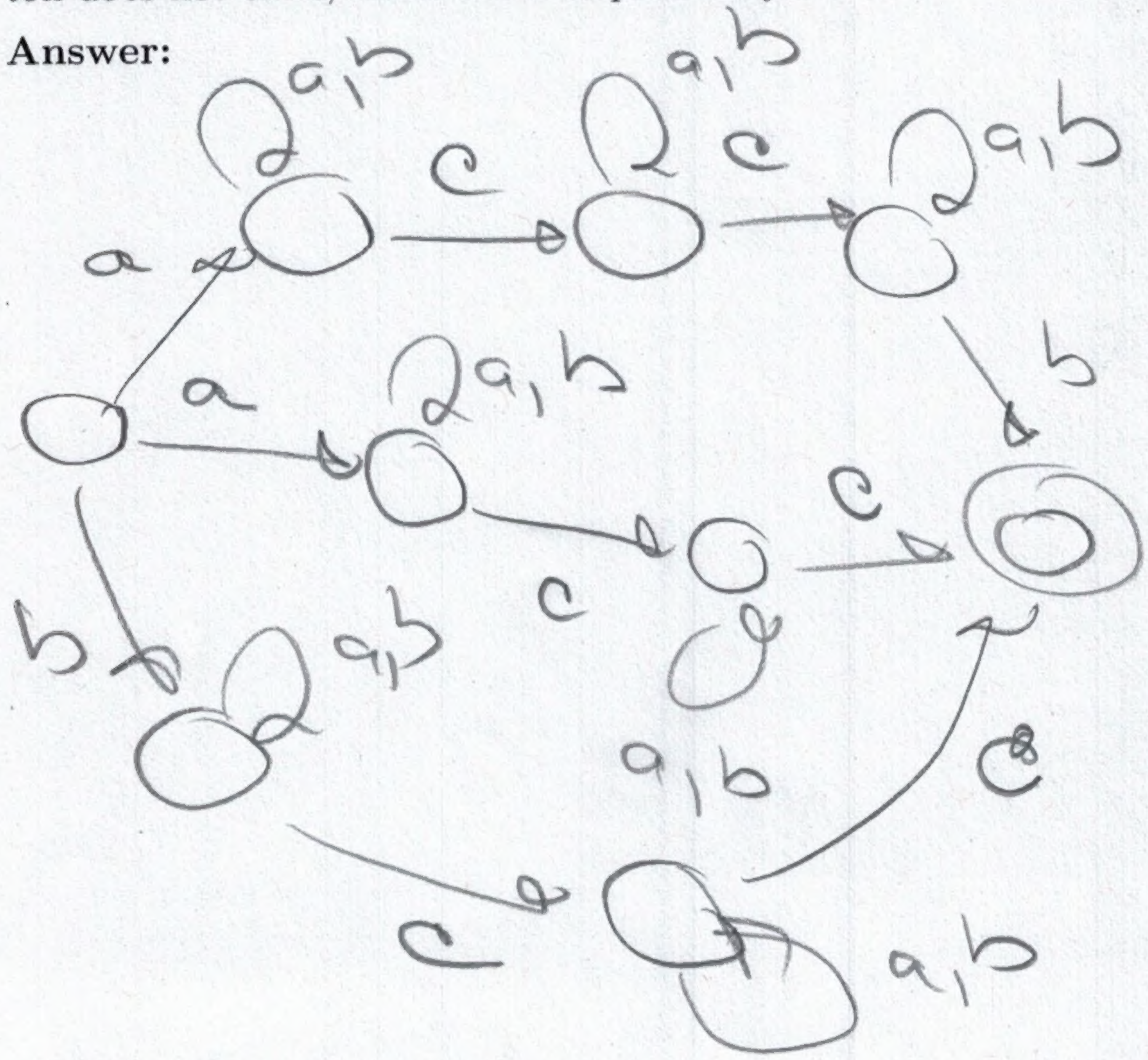
(a) Write a regular expression that represents the language L. If such a regular expression does not exist, state it and explain why.

Answer:

$$a(a\cup b)^*c(a\cup b)^*c(a\cup b)^*b$$
$$\cup$$
$$a(a\cup b)^*c(a\cup b)^*c$$
$$\cup$$
$$b(a\cup b)^*c(a\cup b)^*c$$

(b) Draw a state-transition graph of a finite automaton that accepts the language L. If such an automaton does not exist, state it and explain why.

Answer:



(c) Write a complete formal definition of a context-free grammar that generates the language L. If such a grammar does not exist, state it and explain why.

Answer:

$G = (V, \Sigma, P, S)$

$\Sigma = \{a, b, c\}$

$V = \{S, A, B, D, E\}$

$S \to A \mid B \mid D$

$A \to aEcEcEb$

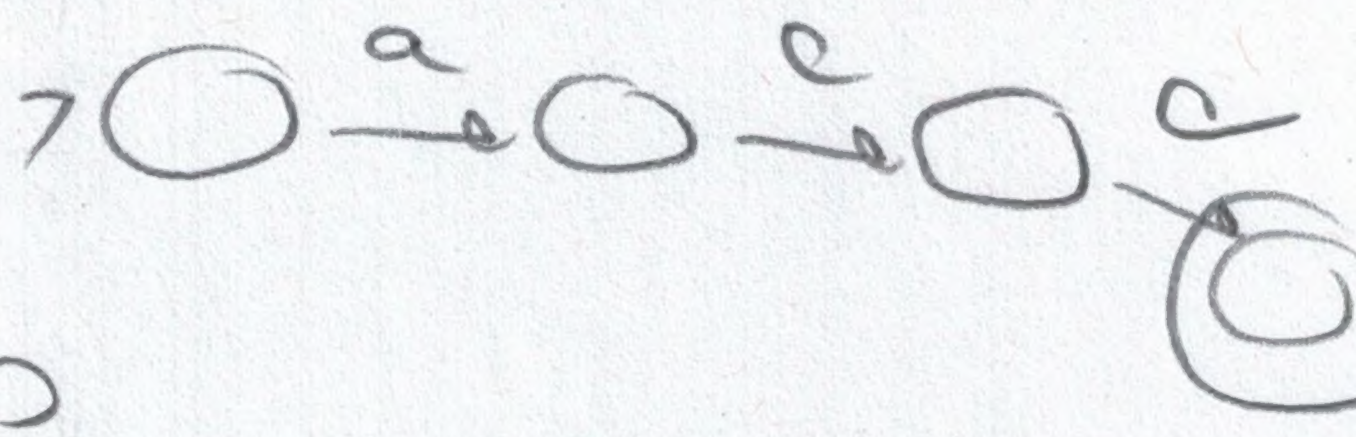
$B \to aEcEc$

$D \to bEcEb$

$E \to \varepsilon \mid EE \mid a \mid b$

(d) Draw a state-transition graph of a finite automaton that accepts the language $L \cap \boldsymbol{a}\,\boldsymbol{c}^*$. If such an automaton does not exist, state it and explain why.

Answer:



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Problem 6 Let L_1, L_2 be languages over the alphabet $\{a, b, c, d, g, e\}$, defined as follows:

$$L_1 = \{g^{3k}\, e^{2i+3}\, d^{2\ell}\, c^{2t+1}\, b^{\ell}\, a^{k}\}$$

$$L_2 = \{c^{2m+3}\, a^{3m+1}\, d^{2n}\, g^{j+2}\, e^{3p}\, b^{j+1}\}$$

where $m, j, n, p, i, k, \ell, t \geq 0$.

(a) Write a complete formal definition of a context-free grammar that generates L_1. If such a grammar does not exist, state it and explain why.

Answer:

$G = (V, S, P, T_1)$

$\Sigma = \{a, b, c, d, g\}$

$V = \{T_1, A, B, D\}$

$P:\ T_1 \to ggg\,T_1 a \mid AB$

$A \to eeA \mid eee$

$B \to ddBb \mid D$

$D \to ccD \mid c$

(b) Write a complete formal definition of a context-free grammar that generates L_2. If such a grammar does not exist, state it and explain why.

Answer:

$G = (V, \Sigma, P, T_2)$

$V = \{T_2, E, F, H, J\}$

$\Sigma = \{a, b, c, d, g\}$

$P:\ T_2 \to EFH$

$E \to ccEaaa \mid ccca$

$F \to ddF \mid \lambda$

$H \to gHb \mid ggJb$

$J \to eeeJ \mid \lambda$

(c) Write a complete formal definition of a context-free grammar that generates $(L_1 \cup L_2)^*$. If such a grammar does not exist, state it and explain why.

Answer:

$G = (V, \Sigma, P, S)$

$V = \{S, T_1, A, B, D, T_2, E, F, H, J\}$

$\Sigma = \{a, b, c, d\}$

$P:\ S \to \lambda \mid SS \mid T_1 \mid T_2$

$T_1 \to ggg\,T_1 a \mid AB$

$A \to eeA \mid eee$

$B \to ddBb \mid D$

$D \to ccD \mid c$

$T_2 \to EFH$

$E \to ccEaaa \mid ccca$

$F \to ddF \mid \lambda$

$H \to gHb$

$H \to ggJb$

$J \to eeeJ \mid \lambda$

(d) Write a complete formal definition of a context-free grammar that generates $L_1^* \cup L_2^*$. If such a grammar does not exist, state it and explain why.

Answer:

$G = (V, \Sigma, P, S)$

$\Sigma = \{a, b, c, d, g\}$

$V = \{S, S_1, S_2, T_1, T_2, A, B, D, E, F, H, J\}$

$P:\ S \to S_1 \mid S_2$

$S_1 \to \lambda \mid S_1 S_1 \mid T_1$

$S_2 \to \lambda \mid S_2 S_2 \mid T_2$

$T_1 \to ggg\,T_1 a \mid AB$

$A \to eeA \mid eee$

$B \to ddBb \mid D$

$D \to ccD \mid c$

$T_2 \to EFH$

$E \to ccEaaa \mid ccca$

$F \to ddF \mid \lambda$

$H \to gHb$

$H \to ggJb$

$J \to eeeJ$

$J \to \lambda$

LAST NAME: ____________________

FIRST NAME: ____________________

Problem 6 Let L_1, L_2 be languages over the alphabet $\{a, b, c, d, g, e\}$, defined as follows:

$$L_1 = \{a^{3m+2}\, c^{2m+1}\, e^{2n}\, b^{j+3}\, g^{3p}\, d^{j+2}\}$$

$$L_2 = \{b^k\, g^{2i+1}\, a^{\ell}\, e^{2t+3}\, d^{2\ell}\, c^{3k}\}$$

where $m, j, n, p, i, k, \ell, t \geq 0$.

(a) Write a complete formal definition of a context-free grammar that generates L_1. If such a grammar does not exist, state it and explain why.

Answer:

$G=(V,\Sigma,P,S)$

$\Sigma=\{a,b,c,d,g\}$

$V=\{S,A,B,D,E\}$

P: $S \to ABD$

$A \to aaaAcc \mid aac$

$B \to eeB \mid \lambda$

$D \to bDd \mid bbbEdd$

$E \to gggE \mid \lambda$

(b) Write a complete formal definition of a context-free grammar that generates L_2. If such a grammar does not exist, state it and explain why.

Answer:

$G=(V,\Sigma,P,T)$

$V=\{T,F,H,J\}$

$\Sigma=\{a,b,c,d,g\}$

P: $T \to bTccc \mid FH$

$F \to ggF \mid g$

$H \to aHdd \mid J$

$J \to eeJ \mid eee$

(c) Write a complete formal definition of a context-free grammar that generates $L_1^* \cup L_2^*$. If such a grammar does not exist, state it and explain why.

Answer:

$G=(V,\Sigma,P,Q)$

$V=\{Q,Q_1,Q_2,S,A,B,D,E,T,F,H,J\}$

$\Sigma=\{a,b,c,d,g\}$

P: $Q \to Q_1 \mid Q_2$

$Q_1 \to \lambda \mid Q_1Q_1 \mid S$

$S \to ABD$

$A \to aaaAcc \mid aac$

$B \to eeB \mid \lambda$

$D \to bDd \mid bbbEdd$

$E \to gggE \mid \lambda$

$T \to bTccc \mid FH$

$F \to ggF \mid g$

$H \to aHdd \mid J$

$J \to eeJ \mid eee$

(d) Write a complete formal definition of a context-free grammar that generates $(L_1 \cup L_2)^*$. If such a grammar does not exist, state it and explain why.

Answer:

$G=\{V,\Sigma,P,Q\}$

$\Sigma=\{a,b,c,d,g\}$

$V=\{Q,S,A,B,D,E,T,F,H,J$

P: $Q \to \lambda \mid QQ \mid S \mid T$

$S \to ABD$

$A \to aaaAcc \mid aac$

$B \to eeB \mid \lambda$

$D \to bDd \mid bbbEdd$

$E \to gggE \mid \lambda$

$T \to bTccc \mid FH$

$F \to ggF$

$H \to aHd$

$H \to J$

$J \to eee$

$J \to eee$

LAST NAME: ____________________

FIRST NAME: ____________________

Problem 7 Let L be the set of exactly those strings over the alphabet $\{a, b, c, d\}$ which satisfy all of the following properties.

1. the string is a concatenation of four non-empty palindromes;
2. three of the four palindromes have an odd length;
3. one of the four palindromes has an even length;
4. the four palindromes may appear in any order;
5. the middle symbol of each of the three odd-length palindromes is different from d;
6. the middle two symbols of the even-length palindrome are different from a.

Write a complete formal definition of a context-free grammar that generates the language L. If such a grammar does not exist, state it and explain why.

Answer:

$G = (V, \Sigma, P, S)$

$\Sigma = \{a, b, c, d\}$

$V = \{S, E, D\}$

P: $S \to EDDD \mid DEDD \mid DDED \mid DDDE$

$E \to aEa \mid bEb \mid cEc \mid dEd \mid bb \mid cc \mid dd$

$D \to aDa \mid bDb \mid cDc \mid a \mid b \mid c$

LAST NAME: __________

FIRST NAME: __________

Problem 7 Let L be the set of exactly those strings over the alphabet $\{a, b, c\}$ which satisfy all of the following properties.

1. the string is a concatenation of four non-empty palindromes;
2. three of the four palindromes have an even length;
3. one of the four palindromes has an odd length;
4. the four palindromes may appear in any order;
5. the middle symbol of the odd-length palindrome is different from a;
6. the middle two symbols of each of the three even-length palindromes are different from d.

Write a complete formal definition of a context-free grammar that generates the language L. If such a grammar does not exist, state it and explain why.

Answer:

$G = (V, \Sigma, P, S)$

$\Sigma = \{a, b, c\}$

$V = \{S, E, D\}$

$P: S \to DEEE \mid EDEE \mid EEDE \mid EEED$

$E \to aEa \mid bEb \mid cEc \mid aa \mid bb \mid cc$

$D \to aDa \mid bDb \mid cDc \mid b \mid c$